STRUCTURAL DESIGN II

05. PLASTIC ANALYSIS

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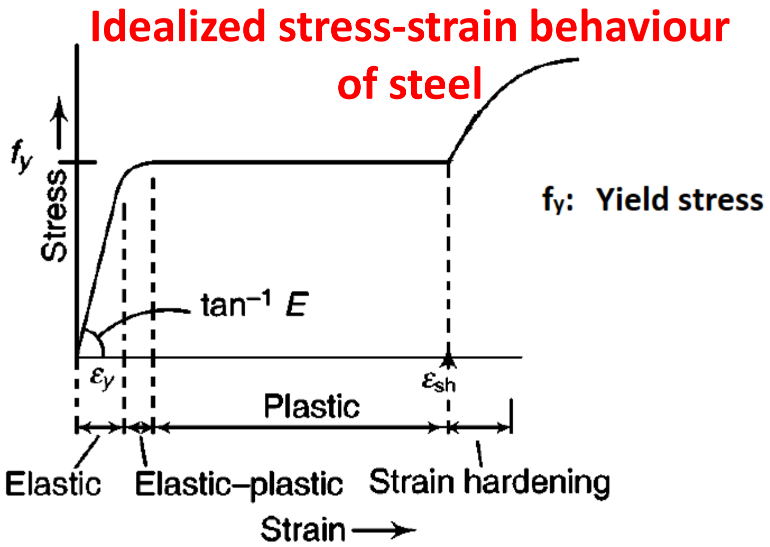
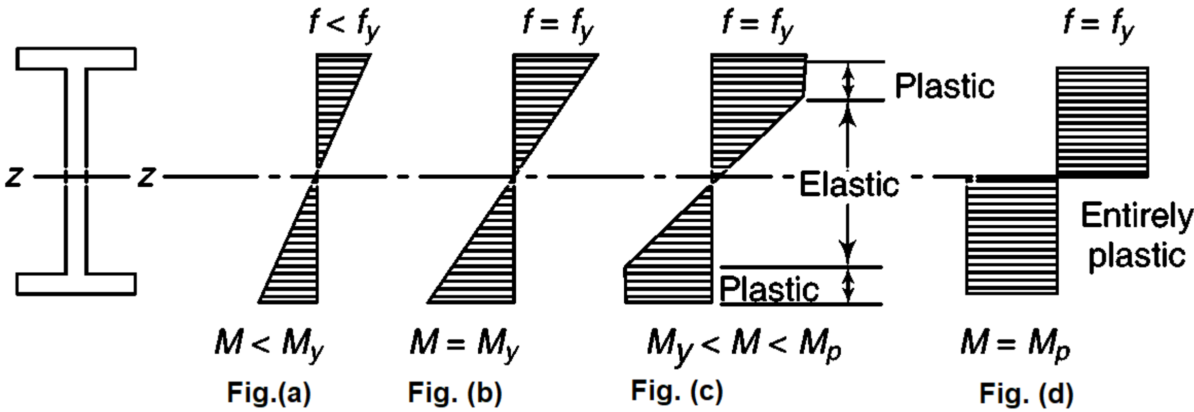
INTRODUCTION



- In **working stress method**,
  - the stress in the material is restricted well within the Yield stress (f_y).
  - Stress-strain relationship is linear and hence simpler for analysis. This region of graph is ideally termed as Elastic Region and hence the analysis is called **Elastic Analysis**.
  - Since the strength of the material beyond the yield point is not utilized, the structures designed using this method is generally heavier, and hence less economical.
- Hence, the method of **Plastic Analysis** is introduced here.
  - Utilizes the strength of material beyond yield point. The material behavior upto strain-hardening region is considered here (see above figure).
  - Instead of yield stress, this method uses ultimate stress as the design criterion. Hence this method is also known as ***Ultimate Load method*** or ***Load Factor Design method***.
  - This is relatively a simpler method for the analysis of Statically Indeterminate Structures.

PLASTIC ANALYSIS

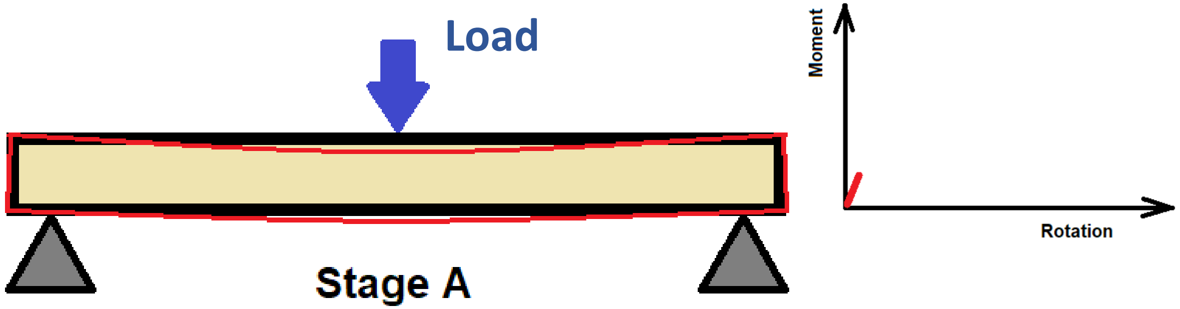


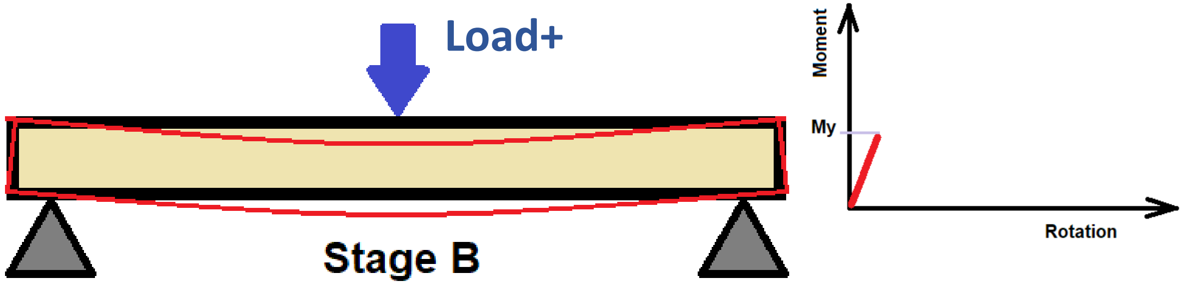
Consider an I-beam subjected to a steadily increasing bending moment.

- **STAGE A:** In the service load range, the section is elastic.
- **STAGE B:** The elastic condition exists until the stress at the extreme fibre reaches the yield stress. When the yield stress reaches the extreme fibre, the nominal moment strength of the beam is referred to as the ***yield moment M_y*** and is given by

$$M_y = Z_e \times f_y$$

where, Z_e is the elastic section modulus $= \frac{I}{y_{max}}$, I is the moment of inertia of the section and y_{max} is the maximum distance between extreme fibre and the neutral axis.





PLASTIC ANALYSIS

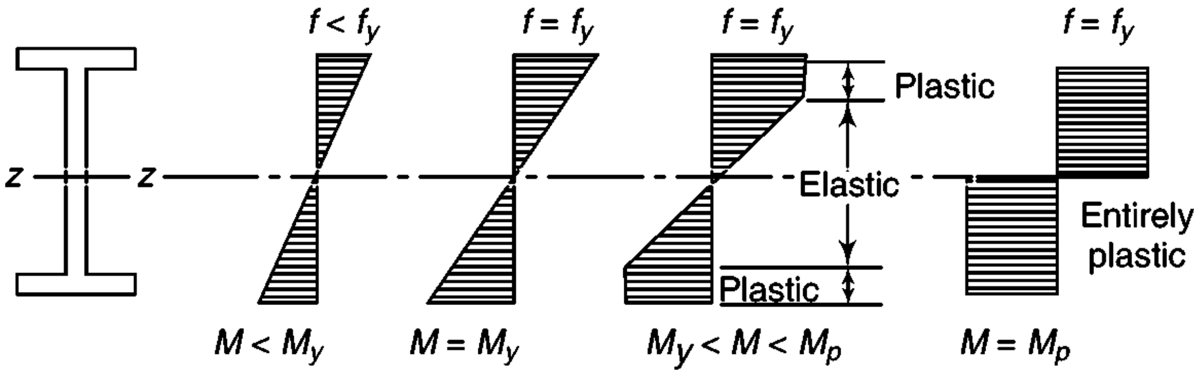
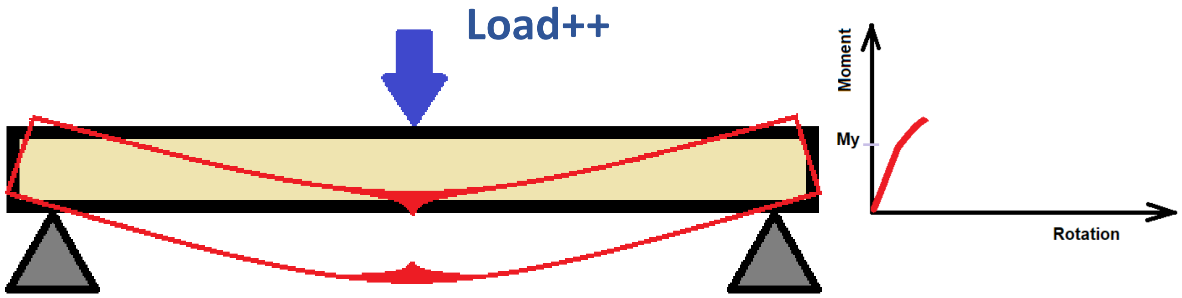


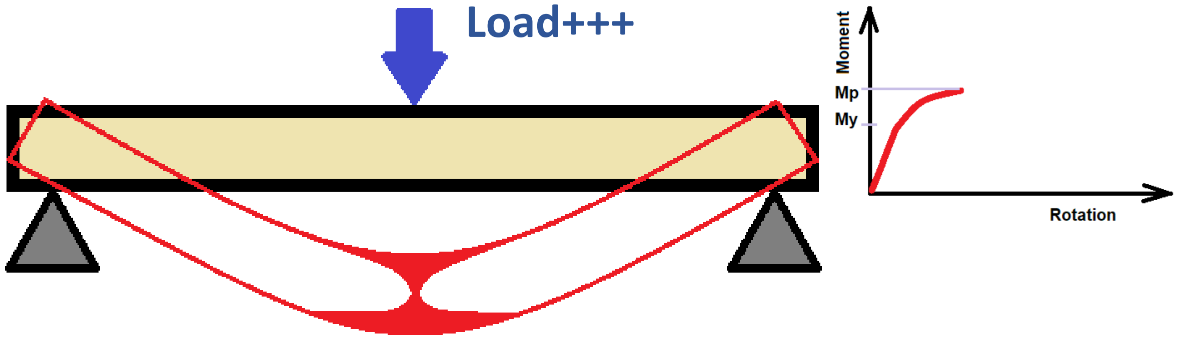
Fig.(a) Fig. (b) Fig. (c) Fig. (d)

- **STAGE C:** A further increase in the bending moment causes the yield to spread inwards from the lower and upper surfaces of the beam. This is because of the yielding of the outer fibres without increase in stresses, as shown by the horizontal line of the idealized stress-strain diagram.
- **STAGE D:** Upon increasing the bending moment further, the whole section yields. When this condition is reached, every fibre has a strain equal to or greater than $\varepsilon_y = f_y / E_s$. The nominal moment strength of the beam at this stage is referred to as the ***plastic moment*** $\mathbf{M_p}$ and is given by

$$M_p = Z_p \times f_y \quad \text{, where } Z_p = \text{plastic section modulus} = \int y \, dA$$

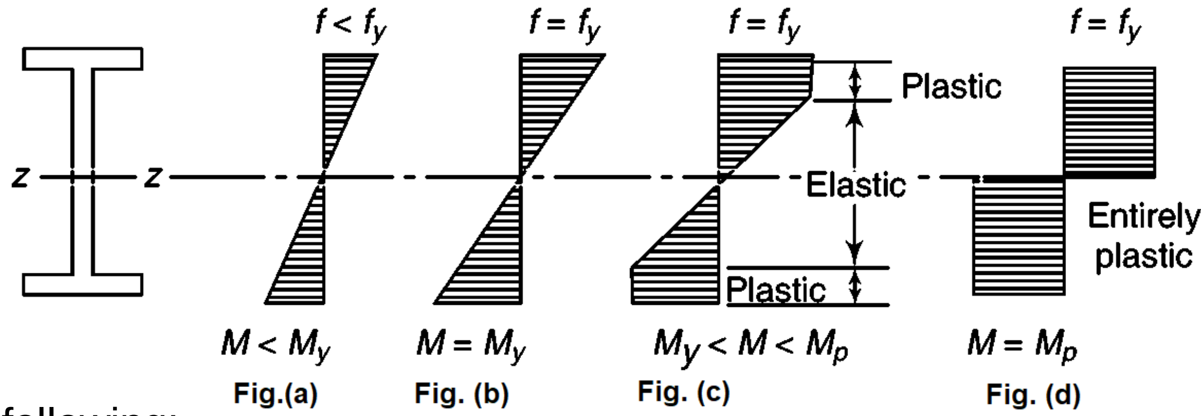


Stage C



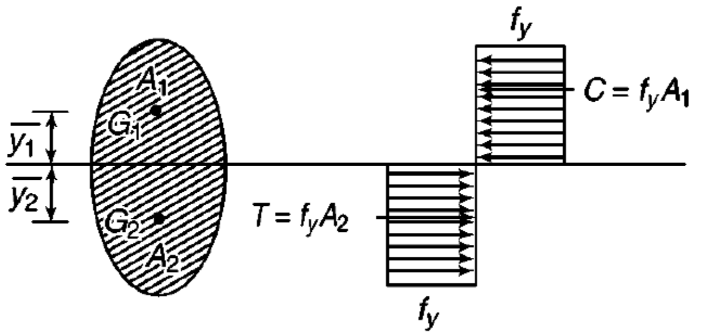
Stage D

PLASTIC ANALYSIS



To determine the Plastic section modulus Z_p, consider the following:

Consider a beam of cross-section area A, has reached Plastic Moment at any section. At equilibrium,



Compressive force C = Tensile force T

=> $f_y A_1 = f_y A_2$ => $A_1 = A_2$

Since $A_1 = A_2$ and $A = A_1 + A_2$,

$$A_1 = A_2 = A/2$$

Plastic moment of resistance

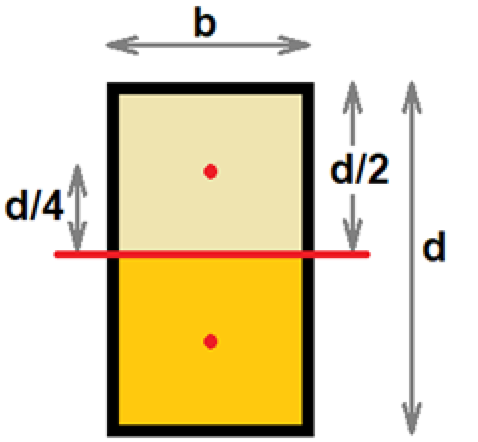
$$M_p = f_y\, A_1\, \bar{y}_1 + f_y\, A_2\, \bar{y}_2$$

$$= f_y\, A/2\; (\bar{y}_1 + \bar{y}_2)$$

Since $M_p = f_y\, Z_p$

=> $\boxed{Z_p = (A/2)\;(\bar{y}_1 + \bar{y}_2)}$ is the plastic modulus of the section.

For a rectangular beam



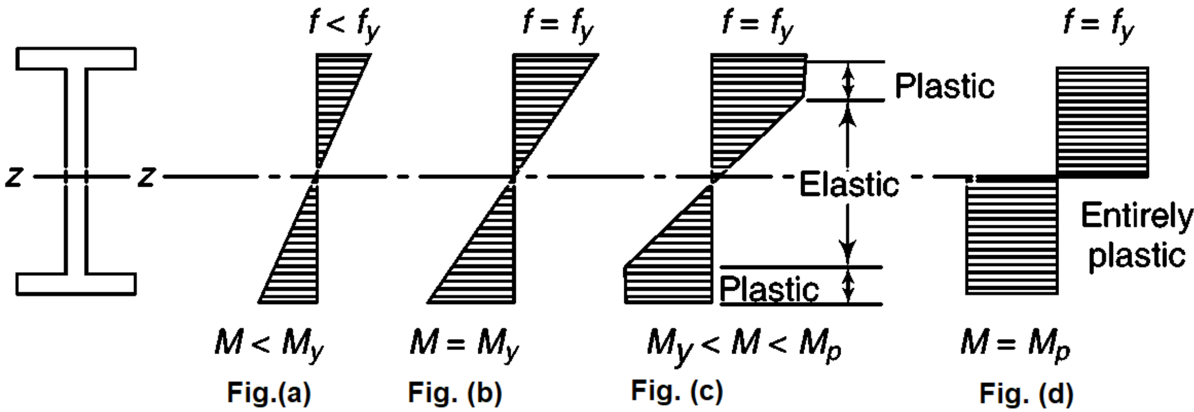
$A = b \times d$

$\bar{y}_1 = d/4$

$\bar{y}_2 = d/4$

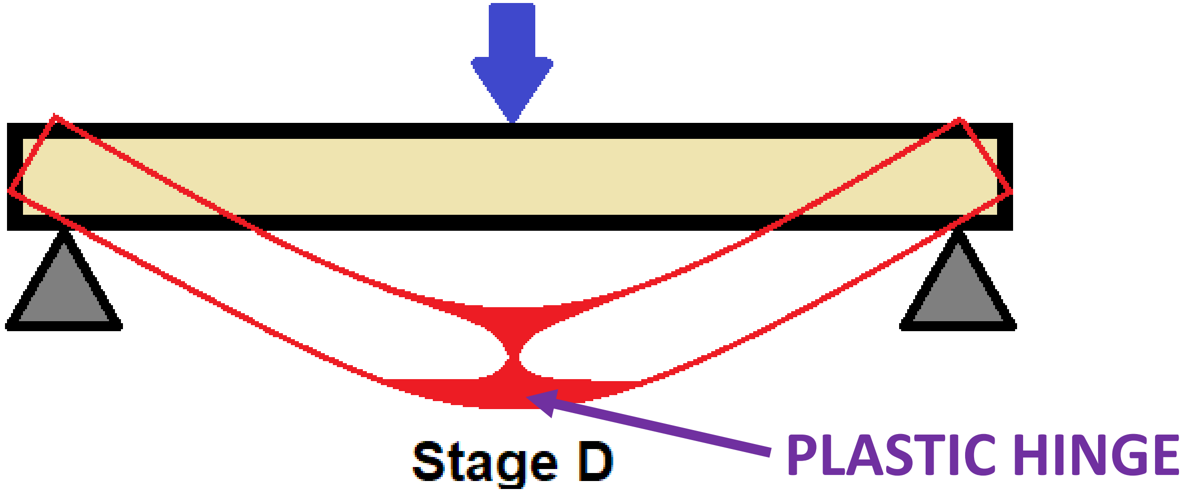
$$Z_p = \left(\frac{bd}{2}\right)\left(\frac{d}{4} + \frac{d}{4}\right) = \frac{bd^2}{4}$$

PLASTIC ANALYSIS

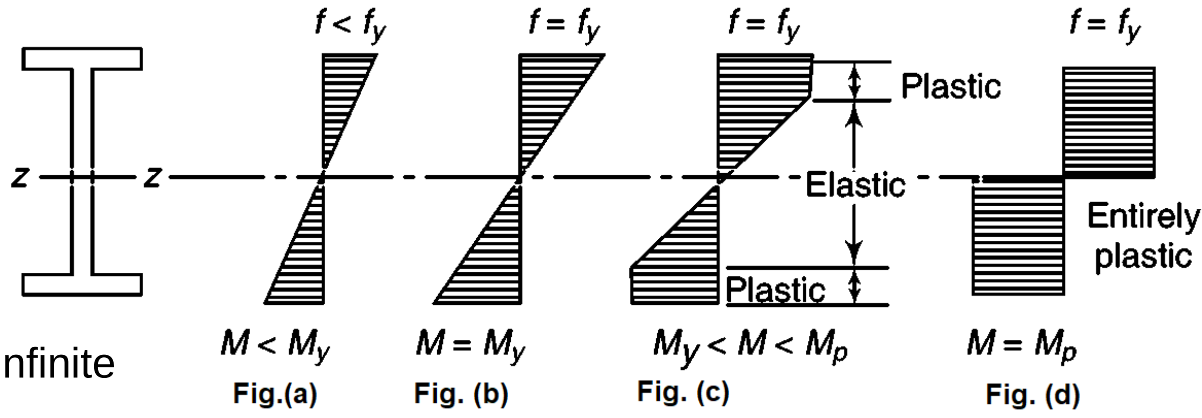


- Any further increase in the bending moment results only in rotation, since no greater resisting moment than the **fully plastic moment M_p** can be developed until strain hardening occurs.

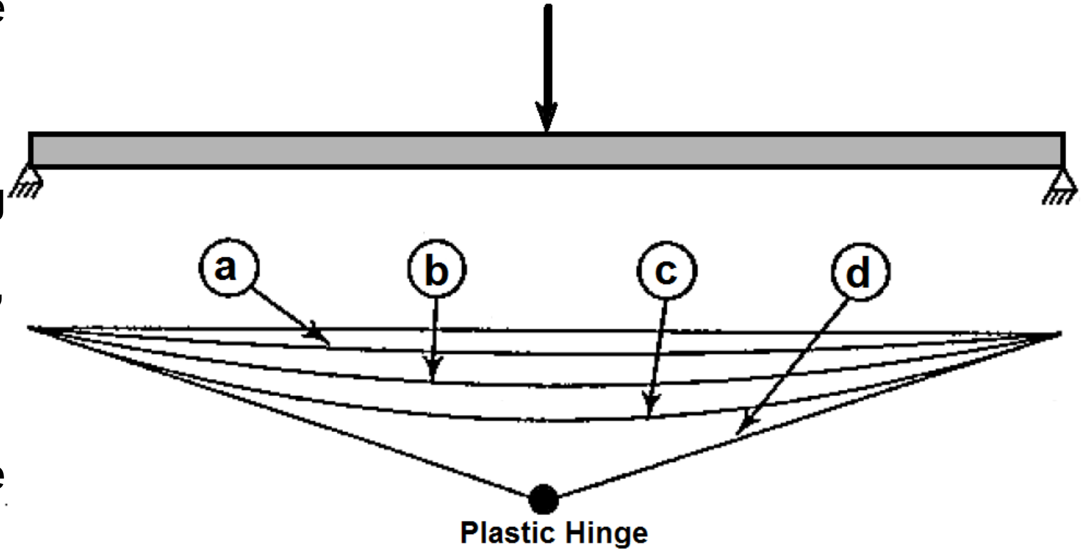
The portion of the member where **M_p** occurs is termed "**plastic hinge**".

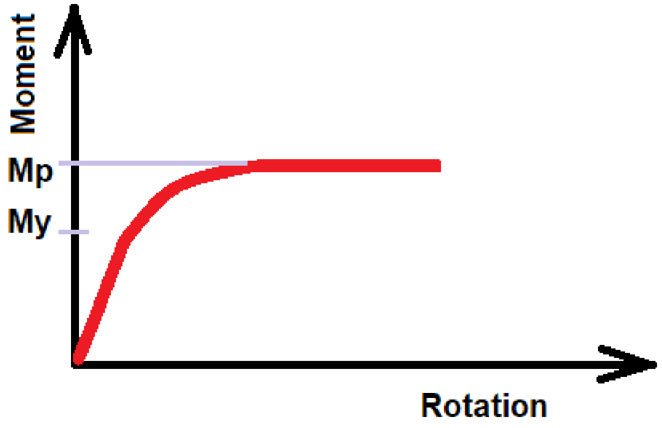


PLASTIC HINGE



- It is defined as a yielded zone, which can cause an infinite rotation to take place at a constant plastic moment $\mathbf{M_p}$ of the section.
- Plastic hinges form in a member at the maximum bending moment locations, such as hinges form at **restrained ends**, **intersections** of members, and beneath **point loads**.
- However, at the intersections of two members, where the bending moment is the same, a hinge forms in the weaker member.
- Generally, No. of plastic hinges are **n = r + 1**, where r is the degree of static indeterminacy.



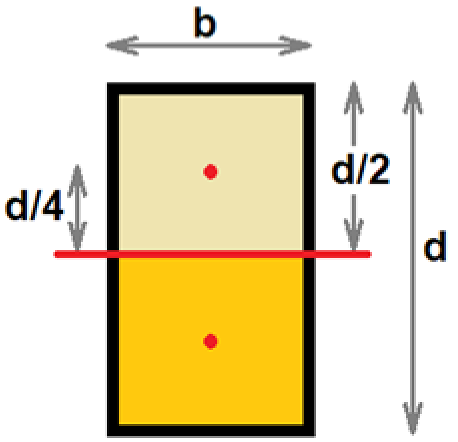


SHAPE FACTOR

- It is a property of the cross-sectional shape and is independent of the material properties.
- It is the ratio of plastic moment to yield moment.

$$S = \frac{M_p}{M_y} = \frac{Z_p}{Z_e}$$

- Indicates reserve capacity of a section (i.e., beyond yielding at extreme fibres to full plastification)
- A section with higher shape factor gives a longer warning before collapse, because such a section is more ductile and gives greater deflection at collapse.



$A = b \times d$

$\bar{y}_1 = d/4$

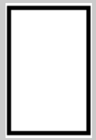
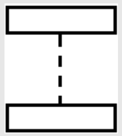
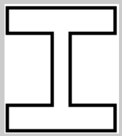
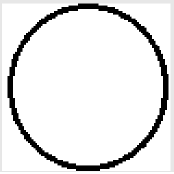
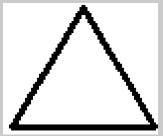
$\bar{y}_2 = d/4$

$$Z_p = \left(\frac{bd}{2}\right)\left(\frac{d}{4} + \frac{d}{4}\right) = \frac{bd^2}{4}$$

$$Z_e = \frac{I}{y} = \frac{(bd^3/12)}{(d/2)} = \frac{bd^2}{6}$$

Shape Factor S

$$= Z_p / Z_e = \frac{6}{4} = 1.5$$

Cross-section	Shape Factor
	1.5
	1
	1.15
	1.7
	2.34

Note that when the material at the centre of the section is increased, shape factor also increases.

LOAD FACTOR

- Ratio of Collapse load to Working Load.

$$\boldsymbol{F} = \frac{\mathrm{P}_u}{P}$$

- For a simple beam, where bending moment varies linearly with the load,

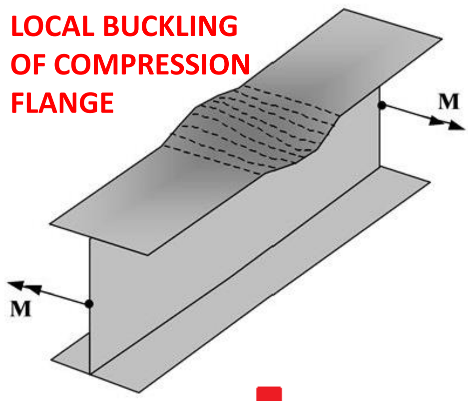
$$\boldsymbol{F} = \frac{\mathrm{P}_u}{P} = \frac{\mathrm{M}_p}{M} = \frac{\mathrm{Z}_p \mathrm{f}_y}{\mathrm{Z}_e f} = \boldsymbol{S}\,\frac{\mathrm{f}_y}{f}$$

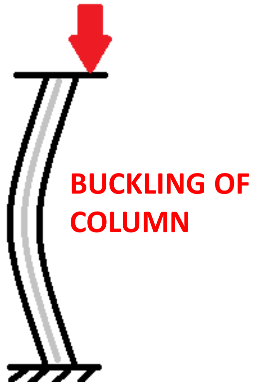
- F varies from 1.7 to 2.

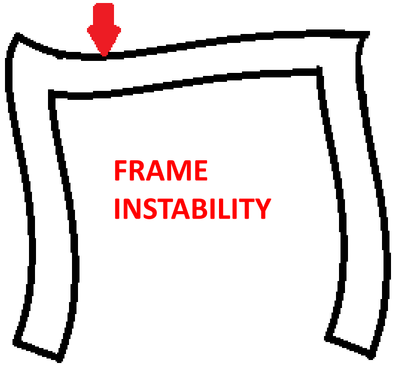
LOCAL BUCKLING OF PLATES

- **Buckling** is defined as a structural behaviour in which deformation develops in a direction or plane perpendicular to that of the loading which produces it.
- Such deformations increase rapidly with increase in the magnitude of the applied loading.
- Occurs mainly in members subjected to compressive forces.
- As a result, changes in geometry can occur in three modes**,**

1) **Local buckling or Local Instability:** Deformation of the cross section of a member. **<= WE RESTRICT OUR CURRENT DISCUSSION TO THIS MODE ONLY**
2) **Buckling of Column or Member Instability:** Deflections/displacements along the length of the member.
3) **Frame Instability:** Overall change of geometry of the structure, causing the joints to displace relative to each other, such as the sway deformation in multi-storeyed frames.



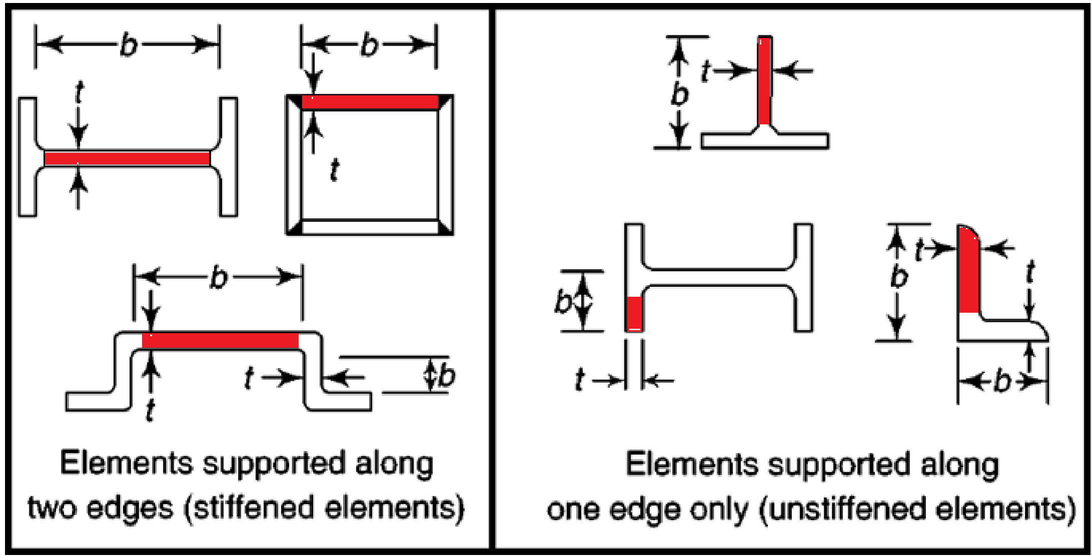




LOCAL BUCKLING OF PLATES

- **Local Buckling causes premature failure of beam and hence should be avoided, otherwise the beam section shall not be utilized upto its plastic moment capacity**.

- In the study on **Local Buckling**, any structural steel section may be considered to be composed of plate elements. These plate elements can be divided into two categories
    1) **Unstiffened elements**
    2) **Stiffened elements**



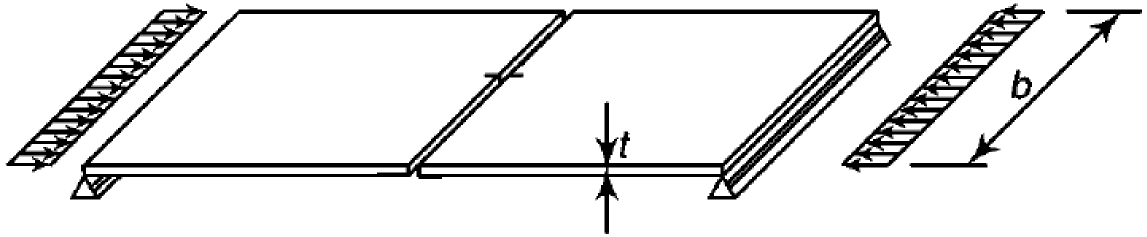
- In beams, local buckling of compression flange (an unstiffened plate element) of the I-section reduces the rotation capacity of the beam and prevents the formation of plastic hinge, instead causes early failure.

Therefore, we make a major assumption in Plastic Theory that the beam is supported continuously laterally to prevent the failure of compression flange by local buckling.

LOCAL BUCKLING OF PLATES

- Critical Local Buckling stress in a plate element is given by:

$$\boldsymbol{f_{cr}} = \frac{K\pi^2 E}{12(1-\mu^2)\left(\frac{\boldsymbol{b}}{\boldsymbol{t}}\right)^{\boldsymbol{2}}}$$



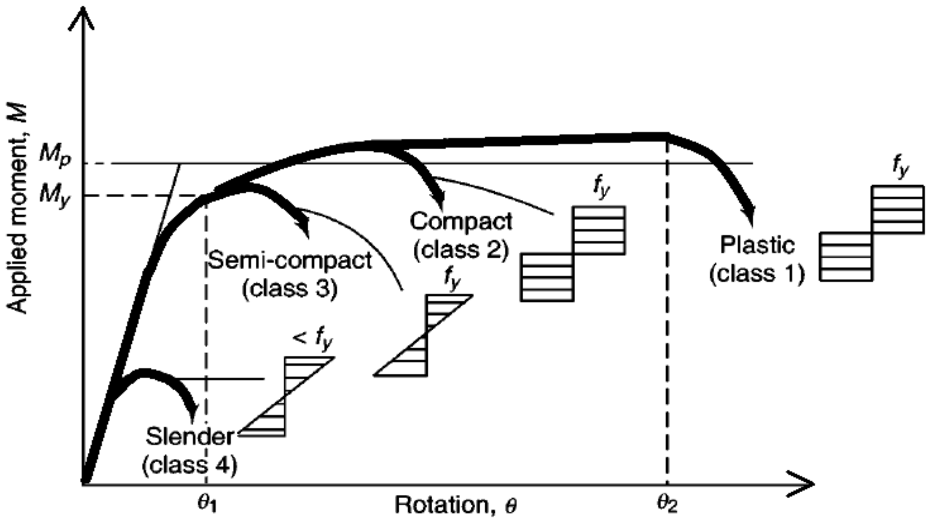
where μ is Poisson's ratio of the material, b/t is the width-to-thickness ratio of the plate, *K* is the buckling coefficient, and *E* is Young's modulus.

- Since f_{cr} is inversely proportional to (b/t) ratio, **Local buckling of plate elements can be prevented by choosing those having smaller value of width-to-thickness ratio**.

CLASSIFICATION OF CROSSSECTIONS

- As per IS 800-2007 **Cl. 3.7 – Page 17,**

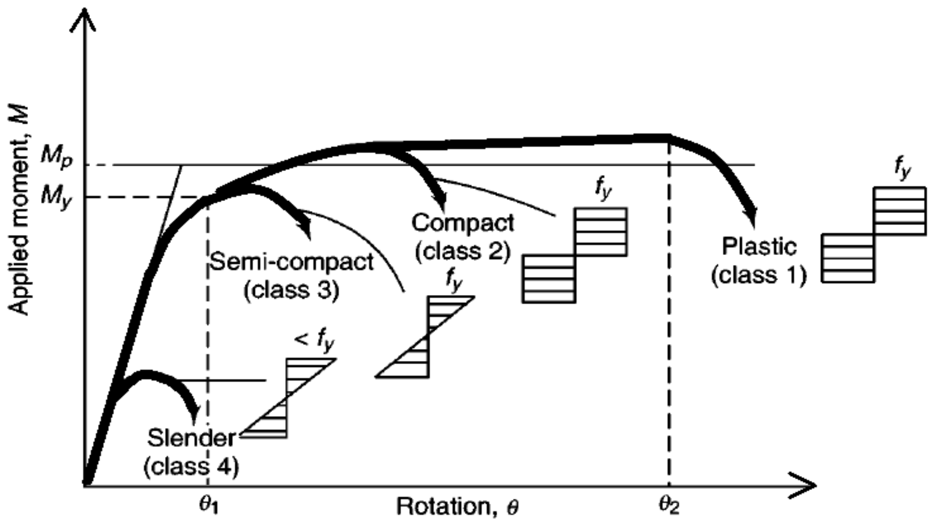
  Cross sections are placed into four behavioural classes depending upon
  - material yield strength,
  - width-to-thickness ratios of the individual components (e.g., webs and flanges),
  - loading arrangement, and
  - capacity to form plastic hinges.
- The four classes of sections are defined as follows:
  1) Class 1 (Plastic)
  2) Class 2 (Compact)
  3) Class 3 (Semi-compact)
  4) Class 4 (Slender)



CLASSIFICATION OF CROSSSECTIONS

1) *Plastic* or *Class 1* Cross sections

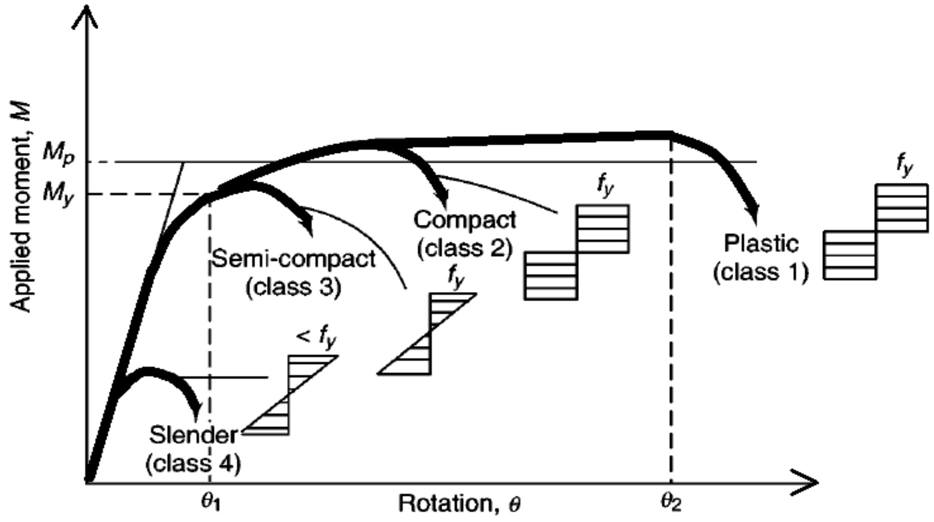
- They can develop Plastic Moment (i.e., Maximum Moment = M_p) as well as sufficient rotation capacity (i.e., $\Theta_2 > 6\Theta_1$; where Θ_1 is the rotation at the onset of plasticity and Θ_2 is the minimum rotation required to qualify as a plastic section)
- Thus, they can develop Plastic hinges
- Hence, fully effective under pure compression
- only these sections are used in plastic analysis and design



CLASSIFICATION OF CROSSSECTIONS

2) Compact *or* ***Class 2*** **Cross sections**

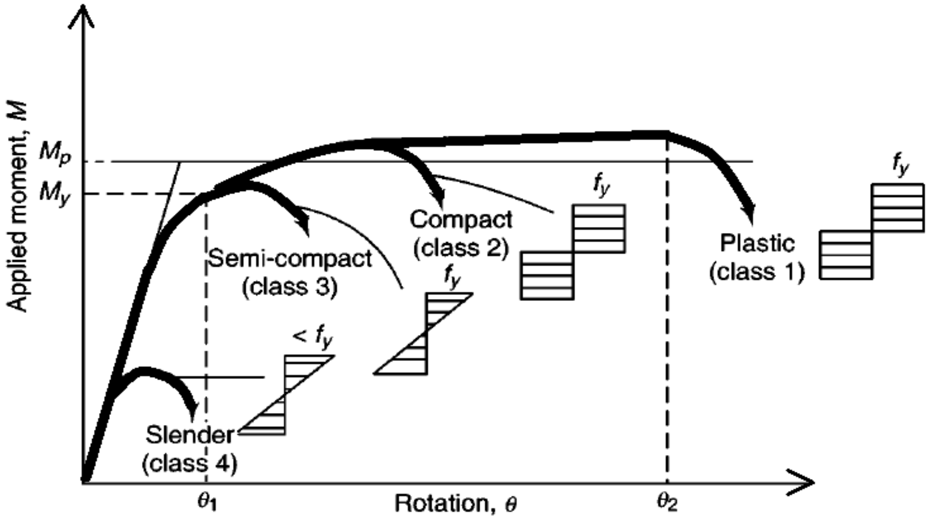
- They can develop Plastic Moment (i.e., Maximum Moment = M_p) but cannot attain sufficient rotation capacity (i.e., $\Theta_2 > \Theta_1$ but $\Theta_2 < 6\Theta_1$)
- Thus, they cannot develop Plastic Hinges, because of local buckling.



CLASSIFICATION OF CROSSSECTIONS

3) ***Semi-compact*** *or* ***Class 3*** **Cross sections**

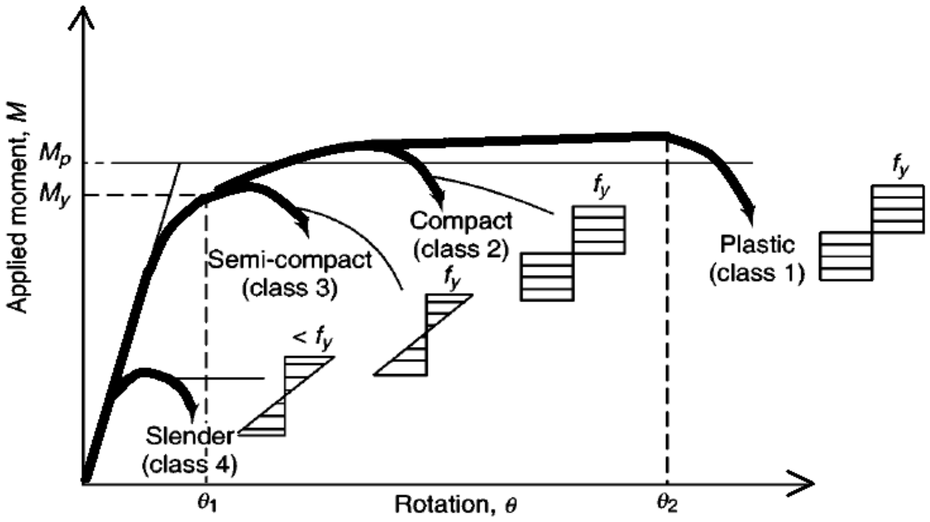
- They can develop Elastic Moment (i.e., Maximum Moment = M_y) only.
- Thus, at that section, only the extreme fibres reach yield stress. This is because local buckling prevents the development of the plastic moment.



CLASSIFICATION OF CROSSSECTIONS

4) ***Slender*** *or* ***Class 4*** **Cross sections**

- Local buckling will occur even before the attainment of yield stress in one or more parts of the cross section.
- They cannot even develop Elastic Moment (i.e., Maximum Moment < M_y).



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